VOL. 14, NO. 10 AIAA JOURNAL OCTOBER 1976

Unsteady Two-Dimensional Flow Using Dowell's Method

Paul H. Park*
Princeton University, Princeton, N. J.

Nomenclature

$C_p \equiv 2p/\rho_\infty U_\infty^2$	= pressure coefficient due to airfoil motion
$\Delta C_p \equiv 2C_p$	= total pressure difference coefficient for a symmetric airfoil
C_L	= coefficient of lift due to airfoil motion
C_M	= coefficient of moment due to airfoil motion
$C_{p,\alpha}$; $C_{p,h/c}$; $C_{L,\alpha}$; $C_{L,h/c}$; $C_{m,\alpha}$; $C_{M,h/c}$	= coefficients/mode amplitudes; aerodynamic derivatives
c	= airfoil chord
h	= rigid body vertical translation amplitude of airfoil
Im	= imaginary part
i	$\equiv (-1)^{1/2}$
k	$\equiv \omega c/U_\infty$
M_∞	= freestream Mach number
p	= perturbation pressure
Re	= real part
t	= time
U_∞	= freestream velocity
w	= downwash
x, y, z	= spatial coordinates
α	= rigid body pitch amplitude (angle of attack)
ϕ	= velocity potential
$\Phi_{(\ )}$	= phase angle of subscripted quantity with respect to airfoil-motion $\equiv \tan^{-1}[\mathrm{Im}(\ )/\mathrm{Re}(\ )]$
τ	= airfoil (maximum) thickness to chord ratio
ρ_∞	= freestream density
ω	= frequency of airfoil oscillation
Subscript	
x	$= d/dx$
Symbols	
$\vert\ \vert$	= magnitude of complex quantity

Theme

A method recently developed by Dowell for lifting transonic flows is applied to the two-dimensional flow over an oscillating airfoil. The method has the advantages that 1) its application is only moderately more complicated than the classical theories for subsonic and supersonic flow and 2) its accuracy can be assessed systematically by computing a correction term. No empiricism beyond that inherent in an inviscid, potential flow model is involved. Comparisons with experimental and other theoretical data are encouraging and show that the method is reasonably accurate for shock-free flows.

Content

In recent years, substantial research has been done toward obtaining more accurate solutions of the small perturbations velocity potential equation for isentropic flow around airfoils at transonic speeds. At Mach numbers close to one the usual order of magnitude assumptions used to linearize the potential equation are no longer valid, and different solution methods are needed. Landahl,[1] among others, has derived a simplication of the full nonlinear equation that is valid for isentropic transonic flows but still retains one nonlinear term. Several methods have been devised for the solution of the equation. Reference 2 gives a concise summary of several. A new method has been developed by Dowell[3] which is similar in outlook to the local linearization method developed by Stahara and Spreiter.[4] However, the former method is more straightforward and has the capability to be made increasingly accurate.

Dowell has outlined his method for two- and three-dimensional, unsteady flow.[3] Actual calculations of pressure derivatives for steady, two-dimensional sonic flow were done

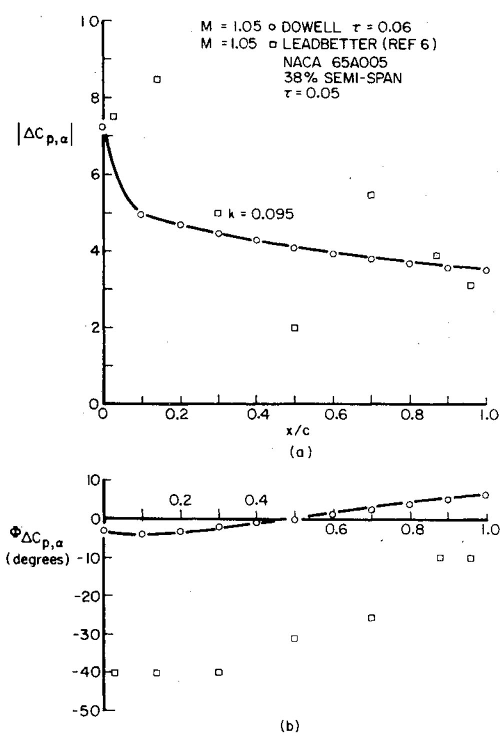

Fig. 1 Pressure derivative magnitude and phase angle versus chord position for parabolic arc airfoils oscillating in pitch about the mid-chord.

Received Dec. 10, 1975; synoptic received May 17, 1976. Full paper available from National Technical Information Service, Springfield, Va., 22151, as N-76-24154 at the standard price (available upon request). The author wishes to express his appreciation to his advisor, Earl Dowell, for his helpful advice and comments.

Index categories: Subsonic and Transonic Flow; Nonsteady Aerodynamics.

*Assistant-in-Research, Department of Aerospace and Mechanical Sciences.

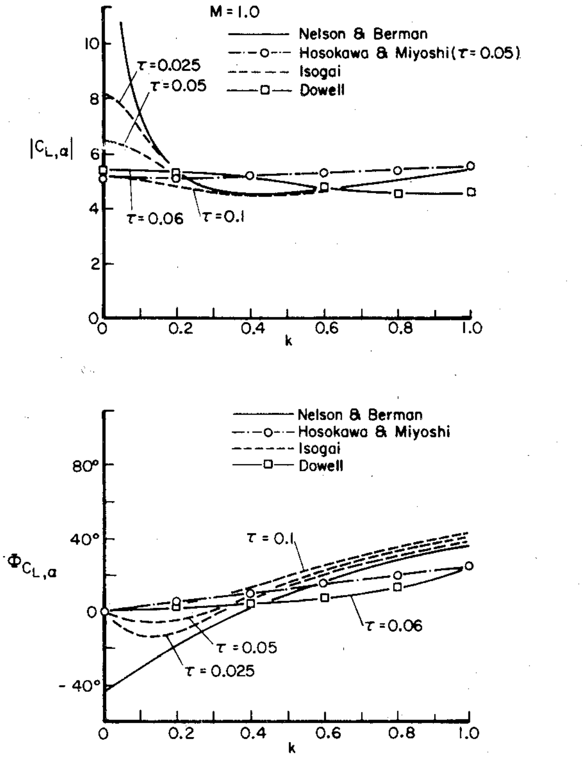

Fig. 2 Lift derivative magnitude and phase for circular-arc airfoils oscillating in pitch about the leading edge at $M = 1.0$ (Ref. 7).

for Guderley and parabolic arc airfoils and they compare favorably with experiment. The method allowed a completely analytical solution, and numerical calculations were made by slide rule.

In Ref. 3, Dowell outlined the solution procedure for calculating pressure derivatives in unsteady two-dimensional sonic flow. The actual calculation of these derivatives along with lift and moment derivatives for a parabolic arc airfoil in heave and pitch is the subject of this paper. Solutions are presented for various reduced frequencies, $k = \omega c / U_\infty$, and, where possible, are compared with other theories and experimental data. The derivation of the Dowell method is carried out in detail in Refs. 3 and 5.

Airfoil motion parameters. The two airfoil motions of most interest in two dimensions are rigid body translation (heave) and pitch. The mathematical expressions for these two modes are as follows

Heave $w(x, t) = w(x)\, e^{i\omega t}$

$$w(x) = i\omega h$$

Pitch $w(x) = [i\omega x - U_\infty]\alpha$

Calculation of pressure derivatives. Figure 1 compares the theory to experimental values by Leadbetter for a 5% thick symmetric airfoil oscillating in pitch about its midchord.[6] Good agreement is seen for the magnitudes, especially near the trailing edge. The phase angles agreed in trend, but differed significantly in quantitative terms. Given the somewhat different airfoil shape, the agreement is satisfactory. Comparisons with other experimental and theoretical data are given in Ref. 5

Calculation of lift and moment derivatives. $|C_{L,\alpha}|$, $|C_{L,h/c}|$, $|C_{M,\alpha}|$, $|C_{M,h/c}|$, and their phase angles vs k are given in Ref. 5. The lift derivatives for pitch are compared with three other theories in Fig. 2. The theory of Nelson and Berman is the classical "sonic theory"; Hosokawa and Miyoski's method is similar to Dowell's except that the ϕ_{xx} term in the small perturbation transonic equation is neglected,† and Isogai's method is a "corrected" local linearization procedure. All curves plotted from Dowell's method are in the range of the other theories, and the general trends are the same. For $k \leq 0.5$ Dowell's results correspond closely with Hosokawa's except for $|C_{M,\alpha}|$. This is not surprising due to the similarity of the methods for $M_\infty \equiv 1$. Tiepel[8] has developed independently a method of analysis similar to that of Hosokawa.

Dowell, in Ref. 3, has done preliminary work on applying the method to three-dimensional flow regimes. Further analytic development and actual calculations for these regimes are needed, and this is the logical next step in the development of the method. Also, following the ideas of Eckhaus,[9] one can in principle incorporate the effect of shock waves in the method.

The encouraging results obtained to date for two-dimensional, unsteady flow neglecting shock waves strongly suggest these extensions are worthwhile. The present method provides a very substantial improvement over classical linearized aerodynamic theory with only a modest increase in complexity of the analysis.

References

[1] Landahl, M., *Unsteady Transonic Flow*, Pergamon Press, New York, 1961.

[2] Bland, R., "Comments on NASA Langley Research in Transonic Unsteady Aerodynamics," AGARD Report No. 611, 1973.

[3] Dowell, E. H., "A Simplified Theory of Oscillating Airfoils in Transonic Flow," *Proceedings of the Symposium on Unsteady Aerodynamics*, University of Arizona, Tucson, 1975, pp. 655–680.

[4] Stahara, S. S. and Spreiter, J. R., "Development of a Nonlinear Unsteady Transonic Flow Theory," NASA CR-2258, 1973.

[5] Park, P. H., "Calculation of Aerodynamic Derivatives in Unsteady Two-Dimensional Transonic Flow Using Dowell's Linearization Method," Princeton University, Princeton, N. J., AMS Report, 1238-T, Sept. 1975.

[6] Leadbetter, S. A., Clevenson, S. A., and Igoe, W. B., "Experimental Investigation of Oscillatory Aerodynamic Forces, Moments, and Pressures Acting on a Tapered Wing Oscillating in Pitch at Mach Numbers from 0.40 to 1.07," NASA TN-D1236, April 1962.

[7] Isogai, K., "A Method for Predicting Unsteady Aerodynamic Forces on Oscillating Wings with Thickness in Transonic Flow Near Mach Number 1," National Aerospace Laboratory Technical Report NAL-TR-368T, Tokyo, Japan, June, 1974.

[8] Teipel, I., "Eine Erweiterung der 'Parabolischen Methode' zur Berechnung schallnaher Stroemungen," *Zeitschrift fuer Flugwissenschaften*, Vol. 22, p. 9, 1974, pp. 307–313.

[9] Eckhaus, W., "A Theory of Transonic Aileron Buzz, Neglecting Viscous Effects," *Journal of the Aerospace Sciences*, Vol. 29, June 1962, pp. 712–718.

†Hence, it cannot smoothly merge into subsonic or supersonic flow as M_∞ moves away from 1.